Warning!
This presentation contains mathematical material which may not be suitable for people with math phobia.

Statement of the problem

$\neq$

I’m trying to keep the math simple.

Statement of the goal

$=$

Standardization

Standardization is the process of adjusting measurements taken from one device to better match measurements taken from another.
Why do instruments disagree?

- Repeatability
- Black level
- Rejection of scattered light
- White level
- Measurement geometry
- Nonlinearity
- Aperture size
- Wavelength alignment
- Bandwidth difference
- Fluorescence

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Potential standardization formula

\[ R_c(\lambda) = R_m(\lambda) + \beta_1(\lambda) + \beta_2(\lambda)R_m(\lambda) \]

Black level correction
White level correction

\[ R_c(\lambda) = R_m(\lambda) + \beta_1(\lambda) + \beta_2(\lambda)R_m(\lambda) \]

Black level
White level
Nonlinearity
Wavelength alignment
Bandwidth difference

Potential standardization sets

- ChromaChecker
- Lucideon Print Standards
Uses for reference materials

- Verify spectro hasn’t changed
- Assess inter-instrument agreement?
- Standardize one instrument to another???

Requirements for a standardization set

- Durable
- Fade resistant
- Opaque
- Similar surface to samples to measure
- Multiple reflectance values

One example – not so good

- Durable
- Fade resistant
- Opaque
- Similar surface to samples to measure
- Multiple reflectance values
- Lots of transitions at all wavelengths

Looks pretty good, right??

Derivatives show less variety
Less tiles, but perhaps enough variety?

Derivatives have a bit more variety

How can we quantify the amount of variety that a set of samples has?

Linear algebra

\[
\begin{bmatrix} 3.0 \\ 4.0 \end{bmatrix} = \begin{bmatrix} 1.00 & 1.00 \\ 1.00 & 1.01 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}
\]

Solution: \( x = -97 \) and \( y = 100 \)

Changing this by 0.01 meant the equation could be solved

Linear algebra

\[
\begin{bmatrix} 3.0 \\ 4.0 \end{bmatrix} = \begin{bmatrix} 1.00 & 1.00 \\ 1.00 & 1.01 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}
\]

Solution: \( x = -97 \) and \( y = 100 \)

The equation can be solved, but the solution is very sensitive.

\[
\begin{bmatrix} 3.01 \\ 3.99 \end{bmatrix} = \begin{bmatrix} 1.00 & 1.00 \\ 1.00 & 1.01 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}
\]

Solution: \( x = -94.99 \) and \( y = 98 \)

The equation is numerically instable.
Quick look back

We expect there might be some numerical instability in this region.

Numerical stability of the ChromaChecker set

- The magnification is huge
- Small measurement errors will go berserk
- Models including bandwidth are worst
- Wavelengths above 600 nm are worst

Is the Lucideon set any better?

Neither set can be used to standardize one instrument to another if bandwidth is part of the model.

Additional sets tested: Behr paint ramps, BCRA tiles, Munsell Color Checker, Pantone primaries, Sherwin-Williams paints

First conclusion

- Neither set can be used to standardize one instrument to another if bandwidth is part of the model.
- Is there a possible set?
- Additional sets tested: Behr paint ramps, BCRA tiles, Munsell Color Checker, Pantone primaries, Sherwin-Williams paints

Best of set found

Error magnification is still a deal-breaker
First conclusion, amended

- It is unlikely to find a standardization set that will allow standardization for instrument bandwidth.

Can we standardize with models that don't include the bandwidth correction?

Standardization sets without bandwidth correction

Second conclusion

- If we use mathematical models that include
  - black level,
  - white level,
  - wavelength shift, and
  - nonlinearity,
- the two potential standardization sets will only work for a range of wavelengths.

Is there any hope?

A set that could work for standardization

- A combination of the Lucideon set and the Munsell Color Checker is numerically stable.
- Munsell is not durable.
- There is hope that a durable and stable standardization set can be made.
Thank you for braving the math!

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